A

(20222)

M.A./M.Sc.-I Sem.

Printed Pages : 4

Roll

11091CV-III

M.A./M.Sc. Examination, Dec.-2021

MATHEMATICS–III

Differential Equation

(GH-1051)

Time : 1½ Hours] *[Maximum Marks : 50*

Note : Attempt questions from all Sections as per instructions.

Section–A

(Very Short Answer Type Questions)

Note: Answer any *two* questions. Each question carries 5 marks. Very short answer is required not exceeding 75 words. 2×5=10

1. Define Adjoint equation and Self adjoint equation.
2. What do you mean by series solution of ODE.
3. What are the cases where Method of Frobenius fails for the solution of ODE.
4. Differentiate between Complete Integral and Integrating factor of a diff. Equation.

5. Write down the condition for p (x), q (x) and r (x) for which the Linear Diff. Eq. have two linear Independent solution

$$p(x)\,y''(x) + q(x)\,y'(x) + r(x)\,y(x) = 0$$

Section–B

(Short Answer Type Questions)

Note: Answer any ***one*** question out of the following three questions. Each question carries 10 marks. Short answr is required not exceeding 200 words. 1×10=10

6. Solve the differential equation

$$(z^2 - 2yz - y^2)\,.\,p + x\,(y+z)\,.\,q = x\,(y-z)$$

and write down the condition that the solution represents a sphere and also evaluate the co-ordinate of the origin of the sphere.

7. Find the complete integral of

$$(x+y)\,(p+q)^2 + (x-y)\,(p-q)^2 = 1$$

8. Prove that

$$ {}_2F_1(\alpha, \beta; \beta - \alpha + 1; -1) = \frac{\lceil(\beta - \alpha + 1) \cdot \lceil\left(\beta/2 + 1\right)}{\lceil(\beta + 1) \cdot \lceil\left(\beta/2 - \alpha + 1\right)} $$

Section–C

(Detailed Answer Type Questions)

Note : Attempt any *two* questions out of the following five questions. Each question carries 15 marks. Answer is required in detail. 2×15=30

9. (a) Find the complete Integral of

$$ p_1 \cdot p_2 \cdot p_3 = z^3 \cdot x_1 \cdot x_2 \cdot x_3 $$

(b) Prove the following Identity

$$ 2x H_n(x) = 2n H_{n-1}(x) + H_{n+1}(x) $$

10. Reduce the equation

$$ (n-1)^2 \frac{\partial^2 z}{\partial x^2} - y^{2n} \cdot \frac{\partial^2 z}{\partial y^2} = n \cdot y^{2n-1} \cdot \frac{\partial z}{\partial y} $$

to canonical form and find its general solution.

11. Let U be a Harmonic function in the Interior of a rectangle $0 \le x \le a, 0 \le y \le b$ in the xy-plane satisfying Laplace equation

$$ \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = 0 $$

with boundary condition

$u(0, y) = 0, u(a, y) = 0, u(x, b) = 0, u(x, 0) = f(x)$

Determine U (x, y).

12. Find the deflection u (x, t) of the vibrating string *with length* π corresponding to zero initial velocity and initial deflection f (x).

$$ f(x) = k(\sin x - \sin 2x) $$

13. (a) Solve

$$ (D^2 - D'^2 - 3D + 3D')\, Z = xy + e^{x+2y} $$

(b) Classify the equation

$$ (1-x^2)\frac{\partial^2 z}{\partial x^2} - 2xy\frac{\partial^2 z}{\partial x \cdot \partial y} + (1-y^2)\frac{\partial^2 z}{\partial y^2} + 2x\frac{\partial z}{\partial x} + 6x^2 \cdot y\frac{\partial z}{\partial y} - 6z = 0 $$