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(20222)
M.A./M.Sc.-I Sem.

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(2)

11092 CV-III

M.A./M.Sc. Examination, December-2021

METRIC SPACES
(GH-1052)

Time : 1½ Hours]

[Maximum Marks : 50

Note : Attempt questions from each sections as per instructions.

Section-A

(Very Short Answer Questions)

Note : Attempt any two questions of this section. Each question carries 5 marks. Very short answer is required.

2×5=10

1. Consider \mathbb{R}^2 and let ρ be a real valued function defined on $\mathbb{R}^2 \times \mathbb{R}^2$ by setting $\rho(x, y) = \min\{|x_1 - y_1|, |x_2 - y_2|\}$ where $x = (x_1, x_2)$, $y = (y_1, y_2)$. Is ρ a metric ?
2. In a discrete metric space (X, d) , prove that every Singleton is an open set.

3. Find the closure of Q the set of rational numbers in (\mathbb{R}, d) where \mathbb{R} is the set of real numbers equipped with the standard metric.
4. Let (X, d) and (Y, ρ) be metric spaces and let $T : (X, d) \rightarrow (Y, \rho)$ be a contraction mapping. Show that T is continuous.
5. Give an example of a subspace of a metric space (X, d) which is complete but not compact.

Section-B

(Short Answer Questions)

Note : This section contains three questions. Attempt any one question. Each question carries ten marks. Very short answer is required. 1×10=10

6. Prove that every complete subspace of a metric space is closed.

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7. Prove that the continuous image of a connected metric space is connected.
8. Let (X, d) be a metric space and let A and B be any subsets of X . Show that :
- (i) A° [the interior of A] is the largest open subset of A <https://www.validcollege.com>
- (ii) $A \subseteq B \Rightarrow A^\circ \subseteq B^\circ$.
- (iii) $(A \cup B)^\circ = A^\circ \cup B^\circ$.

Section-C

(Detailed Answer Questions)

Note : This section contains five questions. Attempt any two questions. Each question carries 15 marks. Answer is required in detail. $2 \times 15 = 30$

9. Let (X, d) be a metric space and let ρ be a real valued function on $R \times R$ defined as $\rho(x, y) = \min \{1, d(x, y)\} \forall x, y \in X$. Show that ρ is a metric on X .

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10. Let $\{G_1, G_2, \dots, G_n\}$ be a finite collection of open subsets of a metric space (X, d) . Then prove that $\bigcup_{i=1}^n G_i$ is an open subset of X . What can you say about $\bigcap_{i=1}^n G_i$? Justify your answer.
11. Let (X_1, d_1) and (X_2, d_2) be metric spaces and let $f: X_1 \rightarrow X_2$ be a mapping from X_1 into X_2 . Show that f is continuous if and only if $f^{-1}(G)$ is an open subset of X_1 whenever G is an open subset of X_2 .
12. Prove that every contraction mapping defined on a complete metric space (X, d) has a unique fixed point.
13. Let (R, d) be a metric space where d is the standard metric defined on the set of Real numbers R . Show that a subset S of (R, d) is compact if and only if it is closed and bounded.

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