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B.C.A. Examination, June-2023

MATHEMATICS-II

(BCA-201)

Time : 3 Hours]

[Maximum Marks : 75

Note : Attempt all the sections as per instructions.

Section-A

(Very Short Answer Questions)

Note : Attempt all the five questions. Each question carries 3 marks. $5 \times 3 = 15$

1. Define cardinality of set with suitable example.
2. If (L, \leq) be a lattice with operation \vee and \wedge then for any $a, b \in L$ show that $a \leq b \Rightarrow a \wedge b = a$
3. If $u = x^y$ then show that

$$\frac{\partial^2 u}{\partial y \partial x} = \frac{\partial^2 u}{\partial x \partial y}$$
4. Define composite. Inverse functions and exponential functions.
5. Evaluate the following integral by changing the order of integration.

$$\int_0^{\infty} \int_x^{\infty} \frac{e^{-x}}{y} dy dx.$$

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[P.T.O.]

(3)

10. (i) If $z = x^2 \tan^{-1} \left(\frac{y}{x} \right) - y^2 \tan^{-1} \left(\frac{y}{x} \right)$ then prove that

$$\frac{\partial^2 z}{\partial y \partial x} = \frac{x^2 - y^2}{x^2 + y^2}$$

- (ii) Find all the maxima or minimum values of function $y^2 + x^2 y^2 + x^4$
11. (i) Draw the Hass diagram representing the Partial Ordering a|b on $\delta = \langle 1, 2, 3, 4, 6, 8, 12 \rangle$
- (ii) Find sets A and B for which $|A| = 5, |B| = 6$ and $|A \cup B| = 9$; What is $|A \cap B|$?
12. (i) Find the angle of intersection of the spheres $x^2 + y^2 + z^2 - 2x - 4y - 6z + 10 = 0$ and $x^2 + y^2 + z^2 - 6x - 2y + 2z + 2 = 0$
- (ii) Show that the length of the shortest distance between the lines.

$$\frac{x-2}{2} = \frac{y+1}{3} = \frac{z}{4};$$

$$2x + 3y - 5z - 6 = 0 = 3x - 2y - z + 3$$

or $(3, \sqrt{6})$

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Section-B

Note : Attempt any two questions.

$7\frac{1}{2} \times 2 = 15$

6. Prove that the lines.

$\frac{x}{1} = \frac{y-2}{2} = \frac{z+3}{3}$ and $\frac{x-2}{2} = \frac{y-6}{2} = \frac{z-3}{4}$ are coplanar also find the point of intersection.

7. Evaluate $\iint_R xy \, dx \, dy$ over the region in the positive quadrant for which $x + y \leq 1$.

8. If $f: A \rightarrow B$ and $g: B \rightarrow C$ are two bijective functions then $g \circ f: A \rightarrow C$ is a bijective function and $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$

Section-C

Note : Attempt any three questions.

$3 \times 15 = 45$

9. (i) Prove that dual of a complemented lattice is complemented.

(ii) If (L, \leq) is a lattice and $a, b, c \in L$ then

(a) $a \wedge (b \vee c) \geq (a \wedge b) \vee (a \wedge c)$

(b) $a \vee (b \wedge c) \leq (a \vee b) \wedge (a \vee c)$

(4)

13. (i) Evaluate

$$I = \int_0^2 \int_0^x \int_0^{x+y} e^x (y + 2z) \, dx \, dy \, dz$$

(ii) Find the Equation of the sphere which passes through the circle $x^2 + y^2 + z^2 - 2x + 2y + 4z - 3 = 0$, $2x + y + z - 4 = 0$ and touch the plane $3x + 4y - 14 = 0$