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(20222)

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Roll No. ...

M.A./M.Sc. -Ist Sem.

11089 CV-III

M.A./M.Sc. Examination, Dec.-2021

MATHEMATICS–I

Algebra

(GH-1049)

Time : 1½ Hours] *[Maximum Marks : 50*

Note : Attempt **all** the Sections as per instructions.

Section–A

(Very Short Answer Questions)

Note : Answer any *two* questions. Each question carries 5 marks. Very short answer is required. 2×5=10

1. Define a normal subgroup of a group. Is $N = \{1, -1\}$ is a normal subgroup of the multiplicative group $G = \{1, -1, i, -i\}$, if yes, show it.

2. Show that the relation of conjugacy in a group G is an equivalence relation on G.

3. Define Euclidean domain.

4. Show that the polynomial $x^3 - 3$ is irreducible over field of rationals.

5. Define solvable group with an example.

Section–B

(Short Answer Questions)

Note : Attempt any *one* question out of the following three questions. Each question carries 10 marks. Short answer is required. 1×10=10

6. If H is a subgroup of a group G and N is a normal subgroup of G, show that $H \cap N$ is a normal subgroup of H.

7. Let S be an ideal of a commutative ring R. Show that the ring of residue classes R/S is an integral domain if and only if S is a prime ideal.

8. Show that any two finite fields having the same number of elements are isomorphic.

Section–C

(Detailed Answer Questions)

Note : Attempt any *two* questions out of the following five questions. Each questions carries 15 marks. Answer is required in detail. 2×15=30

9. Show that a mapping

$f : G \rightarrow G$ defined by $f(x) = x^{-1}\ \forall\ r \in G$ is an automorphism of G if and only if G is abelian.

10. Let p be a prime and m be a positive integer such that p^m divides O(G). Show that there exists a subgroup H of the group G such that $O(H) = p^m$.

11. Show that any ring can be imbeded into a ring with unity.

12. Show that product of two primitive polynomials is again a primitive polynomial.

13. Let p be a prime and $n \geq 1$ be an integer. Show that there exists a field with p^n elements.