11. Verify Cayley-Hamilton theorem of the matrix A' and hence find A^{-1} where

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

12. If $\hat{a}$ and $\hat{b}$ are Unit Vectors inclined at an angle 'A', then prove that

(i) $\sin\left(\frac{A}{2}\right) = \frac{1}{2}|a - \hat{b}|$

(ii) $\cos\left(\frac{A}{2}\right) = \frac{1}{2}|\hat{a} + \hat{b}|$

(iii) $\tan\left(\frac{A}{2}\right) = \frac{|\hat{a} - \hat{b}|}{|\hat{a} + \hat{b}|}$

13. Fvaluate the following integrals :

(a) $\int x^2 \sin x \, dx$

(b) $\int x\sqrt{x+2} \, dx$

(c) $\int \frac{x}{(x^2+2)(x^2+3)} \, dx$

A (Printed Pages 4)

(21222) Roll No. ....

BCA-I Sem.

18005

B.C.A. Examination, Dec.-2022

MATHEMATICS - I

(BCA - 101)

Time : Three Hours] *[Maximum Marks : 75*

Note : Attempt questions from **all** sections as per instructions.

Section - A

(Very Short Answer Type Questions)

Note : Attempt **all** questions of this section. Each question carries 3 marks.

$3\times5=15$

1. Give an example of matrices A, B such that AB=0 but $A \neq 0, B \neq 0$.

2. Verify Rolle's theorem for the function

$f(x)=\sqrt{4-x^2}$ in the interval [−2, 2].

3. Evaluate $\lim_{x \to 0} \frac{\tan x - x}{x^2 \tan x}$.

4. Write the relationship between Gamma and Beta function.

5. Find the characteristic roots of the matrix

$$A=\begin{bmatrix} a & h & g \\ 0 & b & f \\ 0 & 0 & c \end{bmatrix}$$

Section-B

(Short Answer Type Questions)

Note : Attempt any **two** questions out of the following three questions. Each question carries 7½ marks.

7½×2=15

6. Explain $\log_e(1+x)$ in ascending powers of 'x'.

7. Solve by Cramer's Rule :

$$5x-7y+z=11$$

$$6x-8y-z=15$$

$$3x+2y-6z=7$$

8. Prove that Conical text of given capacity will require the least amount of Canvas when the height is $\sqrt{2}$ times the radius of the base.

Section-C

(Long Answer Type Questions)

Note : Attempt any **three** questions out of the following five questions. Each question carries 15 marks. 3×15=45

9. Trace the curve

$$4ay^2 = x\,(x-2a)^2$$

10. If $y=(\sin^{-1}x)^2$ prove that

$(1-x^2)y_2 - xy_1 - 2 = 0$ also prove that

$(1-x^2)y_{n+2} - x(2n+1)y_{n+1} - n^2y_n = 0$