18020

B.C.A. Examination, May-2024

MATHEMATICS-III

(BCA-406)

Time : Three Hours] *[Maximum Marks : 75*

Note : Attempt **all** the sections as per instructions.

Section-A

(Very Short Answer Type Questions)

Note : Attempt all the **five** questions. Each question carries three marks. $3\times5=15$

1. Explain periodic functions with examples.
2. Define Exact-differential equation. Check that the given differential equation is exact :

   $e^y dx+(xe^y+2y)dy=0$.

3. Find the general solution, of the following differential equation.

   $y''+y'-6y=0$

4. Define Polar form of complex number and find argument of $-\sqrt{3}-i$.
5. Find the magnitude of the gradient of the function $f=xyz^2$ at (1, 0, 2).

Section-B

(Short Answer Type Questions)

Note : Attempt any **two** questions out of the **three** questions. Each question carries 7.5 marks. $2\times7.5=15$

6. Explain Limit Comparison Test. Test the convergence of the following series.

   $\Sigma u_n = (n^3+1)^{1/3}-n$

7. Explain the monotonic sequence and Bounded Sequence. Show that

   $\lim \sqrt[n]{n} = 1$

8. (a) Compute real and imaginary part of

$$z = \frac{i-4}{2i-3}$$

(b) Compute Square root of

$z = -1-i$.

Section-C

(Descriptive Answer Type Questions)

Note : Attempt any **three** questions out of the following **five** questions. Each question carry equal marks.

$15 \times 3 = 45$

9. Find the Fourier series expansion for

$$f(x) = x + \frac{x^2}{4}, -\pi \leq x < \pi$$

10. (a) Solve :

$(x \log x)y' + y = 3x^3$

(b) Solve the following equation by finding an integrating factor

$(x^3 + xy^3)dx + 3y^2dy = 0$.

P.T.O.

11. Find the general solution of the following equation :

$y'' - 3y' + 2y = 14 \sin 2x - 18 \cos 2x$.

12. Define the gradient and directional derivatives of a vector function. Find the directional derivative of

$f(x, y, z) = x^2 + y^2 + z^2$ at the point (1, 2, 3) in the direction of $3\hat{i} + 4\hat{j} + 10\hat{k}$.

13. (a) Explain Leibnitz test. Show that the series $\frac{1}{\sqrt{1}} - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{4}} + \ldots\ldots\ldots$ is conditionally convergent.

(b) Define absolute Convergent series. Show that the series

$$1 - \frac{1}{2^3} - \frac{1}{4^3} + \frac{1}{3^3} - \frac{1}{6^3} - \frac{1}{8^3} + \ldots\ldots$$

$$+ \frac{1}{(2n-1)^3} - \frac{1}{(4n-2)^3} - \frac{1}{(4n)^3} + \ldots\ldots$$

is absolutely convergent.